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14. ABSTRACT
The activity discussed intends to increase the capability of Unmanned Aerial Vehicles by incorporating information from a vision sensor into the usual data stream for sensing and control. A particular interest is keeping the implementation simple and the amount of information minimal. It is shown that using only the velocity direction in the camera frame leads to significant improvement in the accuracy of the navigation state estimator. With this improvement, it is then possible to implement on-line parameter estimation techniques to make the controller robust against (a) large modelling errors in the dynamics used to to derive the nominal autopilots, and (b) loss of GPS information during flight.

As the eventual goal is cooperative activity, the effort also looked at using minimal vision information to follow a lead vehicle, and implementations of cooperative observations of ground targets.

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Research in Image-Based Cooperation for Autonomous Conventional Aerial Vehicles

Award No. FA9550-05-1-0296

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FINAL REPORT

31 DECEMBER 2008

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1 Introduction

This document is the final report for the grant titled "Research in Image-Based Cooperation for Autonomous Conventional Aerial Vehicles", issued in May of 2005 by the Air Force Office of Scientific Research. The research was performed during the period May, 2005 through September, 2008.

The overall goal of the research described was to develop a structure in which the information from a vision sensor (generally, a CCD camera) can be incorporated into the information stream from the sensors on an inexpensive aerial vehicle. The purpose of this is to enhance cooperative activities of multiple such vehicles. One of the defining characteristics is that these are conventional airframes; that is, they are fixed-wing aircraft that generate lift in the standard manner.

1.1 Areas of Research

The work described herein is preliminary in nature. A result of this is that it is piecemeal. Three avenues of research were undertaken, with the goal of bringing them together as they reached maturity. In the three years of the effort, solid gains were made in each area, but none was sufficiently advanced to begin to blend with the others.

A final point to be made is that in each area, the attempt was to use the least possible information from the vision sensor. This is in keeping with the desire to maintain a simple and easily-implemented structure, that could be used on minimally-capable hardware. Further, in each case the information use was passive; that is, there is no communication required between the vehicles. While cooperative work with information sharing has been shown to be very effective[1], the load on the computational and communications systems is too great for implementation on the classes of vehicles being considered here.

The three areas of research were

1. Vision-based Tracking

   In this effort, one aircraft follows another, using information only from the vision sensor for guidance. This is a beginning step in cooperative tasks such as cooperative sorties.

2. Cooperative Observation of Partially Occluded Ground Targets

   One of the primary uses of unmanned aerial vehicles will be the observation of ground targets. Most (if not all) of the existing work in this area assumes that the targets are visible from all angles. In this effort, that assumption was not made.

   The first step here was to find an orbit for a single vehicle that kept the target in sight. The measure of merit was the duration of the orbit, which was maximized. The second step, only touched upon before the end of the effort, was to do the problem with a pair of vehicles.

3. Robust Control and On-line Parameter Estimation

The use of vision sensors introduces additional information into the state estimation process. This additional information can be used in at least two ways. First, the additional information will produce a solution closer to the actual states when blended into the standard data fusion filter generally used on the classes of small, inexpensive UAVs envisioned in this report. Second, the information might be taken as explicitly redundant; that is, it might be used as a check on the output of the fusion filter.

In the work reported here, the first approach was used. The resulting estimates of states were then used as the bases for (a) a neural net based approach to robust control, and (b) a scheme to estimate on-line the physical parameters of the UAV.

Each of these areas is discussed in detail below. One or more master of science theses were written in each area. Before discussing these applications, however, the inclusion of vision information in the data fusion filter is addressed.

1.2 Personnel Supported

In addition to the principal investigator, who was supported for between zero and four weeks during each of the three summers during which the grant was active, the grant supported four masters-level graduate students. These were Rajtilok Chakravarty, Jaime Alba-Bohorquez, Joe Bhaktiar, and Tatiana Sviridova. These students were also partially supported by the MAE department. Cooperative work was done by Dale Turner, another master's student, without charge to the grant. Each of these five students wrote a thesis on the topic of research.

At the time of this writing, only one paper has been published as a result of these efforts. This is largely due to the timing; much of the work has only recently finished, so that papers have not yet been written and submitted.

2 Including Vision in the Data Fusion Filter

The basis for much of the work performed under this grant was the inclusion of vision information into the data fusion filter (the exception is the vision based tracking section). Briefly, we recall that the standard instrumentation found on the class of Unmanned Aerial Vehicles in question includes a low-cost inertial measurement system (IMU) and a Global Positioning System (GPS) receiver. The outputs from these devices are blended, in effect using the GPS results to perform on-line calibrations and bias estimates for the IMU.[2] This allows the use of far lighter and less expensive sensors than those needed in the past.

In the past decade, vision systems (generally, Charge-Coupled Device (CCD) cameras) have become widely available and increasingly less expensive. These have been incorporated into many current UAV designs. It is well known that it is theoretically possible to estimate all velocities and orientations using solely the vision information; this is the *Eight-point Theorem*[3]. However, this requires the isolation and tracking of at least eight feature points over several frames. This has been investigated, and despite problems, may be viable in some cases[4].

When additional information is included in the data extraction process, however, the problem is much easier[5]. In the work reported here, the only information extracted from the vision sensor is the velocity direction. As the orientation of the camera is known relative to the UAV, this gives the velocity direction in body frame. In the standard fusion filter, the only direct velocity information is from the GPS, which cannot provide information in the body frame. This leads to a lack of observability, in particular, the inability to accurately estimate sideslip. Using the direction information from the vision sensor eliminates this problem, and renders the system fully observable.

A secondary benefit of including this information is that it is no longer necessary to have a full solution from the GPS receiver. Instead, it is sufficient to use the velocity information from three satellites.

The results of the research show that the addition of velocity direction provides a significant increase in the accuracy of the data fusion filter. Further, the filter maintains a useful estimate even when information from only three GPS satellites is available. These results are reported in Chakravarty and Chichka[6], which is included as Appendix A.

3 Vision-based Tracking

In this effort, one aircraft follows another, using information only from the vision sensor for guidance. It is assumed only that the observing aircraft can extract the silhouette of the craft it is following from the camera. The work reported here predates some recent advances in the field[7].

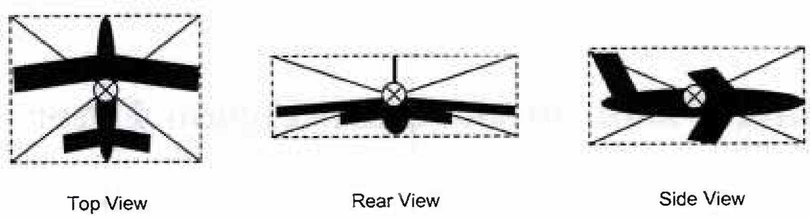

Figure 1: Silhouette in bounding box.

In keeping with the ideal of minimal information, the tracking is performed using only the boundaries of the craft being followed, as in Figure 1. No attempt is made to find the centroid of the actual silhouette; the center of the box in which it appears is sufficient. In Turner[8], it is demonstrated that a simple autopilot can be constructed that uses the size of the bounding box as a range estimate, and the center as a direction guide, that successfully allows a UAV to follow another conventional airframe of similar capability. The reference is included as appendix B.

4 Cooperative Observation of Ground Targets

One of the most likely uses of inexpensive UAVs is observation and reconnaissance. In this effort, we focused on a particular problem that will likely require cooperation between multiple vehicles, that of continuous observation of a partially occluded ground target. While there has been much work done in the past two decades on observation of targets both by a single UAV ([9, 10] and others) and cooperative pairs ([11, 12] and many others), these have involved ground targets that are visible from all angles.

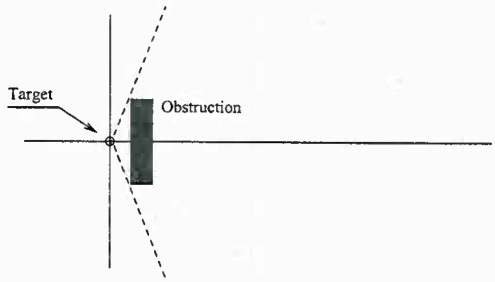

Figure 2: Ground target partially hidden by obstruction.

In this work, we instead consider targets that are partially occluded; that is, they cannot be seen from some angles, as in Figure 2. In the figure, there is a single obstruction, possibly a building. The plane is therefore separated into a "visible" region, from which the target can be seen, and the invisible region. The research undertaken assumed such a division, though it was not assumed that the visible region was the larger.

The research progressed in two steps. The first involved finding an orbit for a single UAV flying at constant altitude and velocity that stayed in the visible region. By piecing together a series of coordinated turns, and assuming that the vehicle could see slightly more than 90 degrees from the nose, such a trajectory was created. The performance index taken was the time of the orbit; the objective was to maximize the time that a single vehicle could maintain observation, so that other UAVs could be used for other tasks. Descriptive parameters were varied to investigate the sensitivity of the orbit duration.

The second step was to extend the methods of Barth[11] to this situation. It was found that the Lyapunov control approach produced viable controller for this situation. Barth used the distances of the vehicles from the target, and from each other, to derive his controller. It was shown in the work reported here that only one vehicle needs to be able to see the target. Full investigation of the controller with this reduced information was not completed.

The full report of this effort is provided in [13], which is enclosed as Appendix C.

5 Analytic Redundancy and On-line Parameter Estimation

Two efforts were made in this area. The first involved neural net augmentation of a linear autopilot. The second used standard parameter estimation techniques to estimate during flight the characteristics of the UAV.

Recall that the standard linear Kalman Filter in discrete time has the form

$$\bar{x}_{k+1} = A_k \hat{x}_k + B_k u_k \tag{1}$$
$$\hat{x}_k = \bar{x}_k + K\left[z_k - H_k \bar{x}_k\right] \tag{2}$$

where the linear system is defined by the relations

$$x_{k+1} = A_k x_k + B_k u_k + w_k \tag{3}$$
$$z_k = H_k x_k + v_k. \tag{4}$$

In this formulation, $x_k \in \Re^n$ is the state, $u_k \in \Re^m$ is the known control, and $z_k \in \Re^p$ is the measurement. The process and measurements are corrupted by w_k and v_k, assumed to be normally distributed and zero mean (any known non-zero mean is easily subtracted out). K is the estimator update gain, the equation for which can be found in any standard reference.

Most importantly in the equations above, the plant, control, and measurement matrices A, B, and H are assumed known in advance for all time steps. This is true for the data fusion filter, as the dynamics consist of integrating the accelerometer measurements, and the IMU measurement noise enters as w. The measurements come from the GPS, and the system is well-known to be (A, H) unobservable.

Were the vehicle dynamics reliably known, it would make more sense to use them in the filter. This is made difficult for two reasons. One is that the linear model is only valid near the nominal (usually straight and level) flight condition, while UAVs often take quite severe maneuvers. The other is that inexpensive UAVs may have very different flight characteristics, even when they are theoretically identical. They also may suffer minor damage in the field.

In this work, two approaches were taken to dealing with this. The first simply augments the nominal linear controller, as in Sharma and Calise[14]. This forces the UAV to act as the nominal UAV would, which is important when the vehicle is operating in tight spaces or near another vehicle. While useful, this approach is limited.

Ideally, we would like to be able to operate not only in tight confines, but when GPS is denied. This is likely to be increasingly important. Thus, the second approach taken attempts to derive the values of the dynamic coefficient matrices on-line, taking advantage of the wealth of information provided by GPS and the vision sensor. Then, when GPS fails or is denied, the dynamics can be used in a traditional filter to maintain a high-quality estimate of the state.

5.1 Neural Net Augmentation

In deriving the controller for a UAV, the best that can be done is to use the nominal system dynamics in the derivation. This creates a controller that will probably work fairly well, however, it is certain that there will be at least some difference between the nominal system and that of any actual UAV.

In controlling a UAV in close quarters or in formation, it is likely that we desire not only a stable controller, but one that produces the actual commanded output, at lease for a subset of the states. Sharma and Calise[14] derived the theory to allow a neural net to augment the nominal linear controller to achieve this. There are obvious restrictions, but not onerous ones.

In their work, however, the example system was very simple. In this effort, Albs-Bohorquez[15] extended this work to the longitudinal dynamics of an aircraft. In simulation, it was possible to force an aircraft (a Cessna 172) to follow nearly precisely the altitude profile commanded by a linear controller derived for an entirely different craft (a NAVION). This reference is included as Appendix D.

5.2 On-line Parameter Estimation

In this subset of the effort, the UAV runs three filters in parallel. The first is the enhanced data fusion filter, including the vision information as in [6]. The second is a real-time parameter estimator, using standard techniques to find values for the aircraft linear coefficients. The third is a standard linear Kalman Filter, which uses the linear models from the second in a state estimator. This state estimator does not use the GPS as a sensor. However, once a reasonably accurate model of the UAV dynamics has been obtained, the Kalman Filter is able to estimate the states with good accuracy based on input from the inertial navigation system and the vision sensor.

While the basic ideas of parameter estimation for a dynamical system are well-known [16], it was not possible to find an instance of on-line implementation for a UAV in the current literature at the beginning of this effort. This is no longer true; results that partly parallel some of the developments under this effort have since appeared in Farrell and Polycarpou [17]. These investigators used the estimates in a feedback loop to update ongoing controls, however; to our knowledge there has been no other effort to use the estimates in this fashion.

In this work, using simulated data, it was possible to estimate the linear coefficients during the first phase of flight. After this, knowledge of GPS was switched off, and the vehicle continued using the estimated plant and control coefficient matrices. No knowledge of these matrices was assumed at the beginning of the flight. Heuristic methods were used to determine when the linear models were sufficiently good.

The results of this effort were promising. The most immediate shortcoming is that the models used were the standard, separated longitudinal and lateral linear dynamics, as in Nelson[18]. While this is standard, and works very well for flight near the straight and level reference condition, it does not allow for kinds of maneuvers we expect from UAVs in tight confines. It leads to a secondary problem in that data during violent maneuvers tends to cause the estimates of linear coefficients to decay.

There are several directions in which this part of the work, in particular, could be extended. Inclusion of some limited, well-understood nonlinear dynamics should be included in the dynamics model. Weighting of the output of the data fusion filter should be included to speed convergence of the dynamics estimator. The results of the effort, and several further suggestions, are in Sviridove[19], which is included as Appendix E.

6 Conclusion

The effort reported included several subsections, each of which achieved at least minor success. Results are reported, primarily in appendices. At least two areas, those of cooperative observation of occluded ground targets, and on-line parameter estimation, deserve much more attention.

The effort reported has resulted in the publication of one paper, and five master's theses. The theses are included as appendices, as they are not readily available without charge, and because summary of the significant results is almost impossible in a several-page executive summary. The paper is included because it is not very long.

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APPENDIX A

Including Vision Information in a Data-fusion Filter

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AIAA Guidance, Navigation, and Control Confere   and Exhibit
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AIAA 2006-6543

Including Vision Information in a GPS/INS Fusion Filter

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Autonomous, unpiloted aerial vehicles are considered an important part of near-term reconnaissance and other missions. This paper considers the use of such vehicles in which GPS and inertial systems are the primary navigations sensors, with a vision sensor also available. The vision sensor is used to approximate a velocity direction, and this measurement is added to the usual GPS/INS fusion filter to aid in the orientation approximation. The gain in accuracy from this addition is considered. Further, the use of this measurement in cases in which there are too few satellites for a GPS position and velocity solution is considered. When three satellites are available, the range and range-rate information combined with the velocity direction provides a reliable solution.

I. Introduction

Unmanned systems require information about the vehicle states such as position, orientation, and so forth to perform adequately. This information is derived from the output of sensors such as Global Positioning System (GPS) receivers, and an Inertial Navigation System (INS). In many cases, particularly for low-cost, expendable vehicles, these are off the shelf sensors, readily available in the market, with poor signal characteristics. Of course, many of the state values cannot be measured directly, due to physical constraints. In the case examined, the available sensors are an INS, a GPS, and a vision sensor. We use an Extended Kalman Filter implementation of the GPS/INS fusion filter here to give us vehicle state estimates of sufficient accuracy.

Data from a simulated aircraft and sensor model was used for the verification of the Extended Kalman Filter.

I.A. Basic Form and Simplifying Assumptions

I.A.1. Instrumentation

The instrumentation available on the UAV includes a 3-axis accelerometer, 3-axis gyro, and a GPS receiver. We also have a vision sensor fixed to the body coordinate frame that gives us velocity direction information. The information from the INS (consisting of the accelerometers and the rate gyros, with some smoothing and signal processing) can be read at a much higher rate than GPS.

Accelerometers: The accelerometers return sensed acceleration at their location, in the accelerometer coordinate frame. This is fixed to the body frame and nominally aligned with it.

Rate Gyros: These are co-mounted with the accelerometers, and share an axis system. They return sensed angular rates at their location.

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GPS: The GPS receiver provides both inertial position and inertial velocity. The position is provided as latitude, longitude and altitude above sea level. The velocity is provided in Northward, Eastward and Downward components.

I.A.2. Assumptions

In implementing the navigation solution, we make some simplifying assumptions. Some of these are due to the quality of the instrumentation that we have, and some due to the nature of the mission for which the flight computer is needed.

Assumption 1.1: Given the level of accuracy of out inertial sensors, the rotation rate of the earth is lost in the noise. Therefore, we will assume the Earth Centered Earth Fixed (ECEF) frame to be inertially fixed. Also the range of operation of the vehicle is sufficiently small that the curvature of the Earth is negligible, and we will assume a flat-Earth approximation to be sufficient.

Assumption 1.2: We assume that the drift rate of the accelerometers and the rate gyros is sufficiently small that it need not be included in modelling them. The estimate of the bias which we assume to be constant in this case, should track a small drift rate.

Assumption 1.3: The inertial measurement unit is mounted sufficiently close to the center of mass of the vehicle that we need not include the lever arm between it and the center of mass in our calculations. Alternatively, as in this case, we can directly track the position of the IMU and base all the relevant calculations on that.

Assumption 1.4: The position of the GPS antenna relative to the center of mass is non-trivial, but known.

I.B. Filter Structure:

This filter will differ from a textbook estimation filter in that the "process" is the state of the system as driven by the readings from the IMU. Therefore, the noise in the IMU measurements takes the place of process noise. It is only the noise in the GPS readings that comes into the filter as measurement noise. This formulation of the GPS/INS filter is standard, and further details are available in,[1] among others.

I.C. Filter States:

In the case of this particular navigation filter implementation, we add the decision to describe the aircraft orientation using quaternions, rather than the more familiar Euler angles. With this decision, we have a sixteen-state filter:

$$\begin{array}{lcl} \mathbf{p} & \longrightarrow & \text{Inertial position} \\ \mathbf{v} & \longrightarrow & \text{Inertial velocity} \\ \mathbf{q} & \longrightarrow & \text{Quaternions} \\ \mathbf{b} & \longrightarrow & \text{Accelerometer biases} \\ \boldsymbol{\varphi} & \longrightarrow & \text{Rate Gyro biases} \end{array}$$

A bias is defined as a constant error in the measurements. Note that we consider the ECEF frame to be the inertial frame. Also in this filter implementation the gravitational effects are taken care of by the constant vector g incorporated in the dynamical model of the system where it is defined as shown below with units of (m/sec^2).

$$\mathbf{g} = \begin{bmatrix} 0 & 0 & 9.81 \end{bmatrix} \tag{1}$$

II. Equations

II.A. Measurement Equations

Let $\mathbf{a}_s$ and $\boldsymbol{\omega}_s$ be the measured accelerations and angular velocities, respectively. We have assumed that these measurements are affected by constant biases and white noise as given in eq. (2) and eq. (3) below, where it is understood that these values are expressed in the body coordinate system.

$$\mathbf{a}_s = \mathbf{a} + \mathbf{b} + \mathbf{w}_a \tag{2}$$

$$\boldsymbol{\omega}_s = \boldsymbol{\omega} + \boldsymbol{\varphi} + \mathbf{w}_\omega \tag{3}$$

The unsubscripted values are truth values; those subscripted with s are the sensed values. The terms $\mathbf{w}_a$ and $\mathbf{w}_\omega$ are zero-mean with noise processes of known variances; $\mathbf{w}_a \sim N(0, V_a)$ and $\mathbf{w}_\omega \sim N(0, V_\omega)$

The GPS measurements of position and velocity are likewise subject to measurement noise, but we assume that they have zero bias. Thus,

$$\mathbf{p}_G = \mathbf{p}^G + \mathbf{w}_p \tag{4}$$

$$\mathbf{v}_G = \mathbf{v}^G + \mathbf{w}_v \tag{5}$$

The subscript G specifies that the measurement is made by the GPS unit.

II.B. Dynamical System Equations:

The equations of motion for these variables are

$$\dot{\mathbf{p}} = \mathbf{v} \tag{6}$$

$$\dot{\mathbf{v}} = \mathbf{a}^{\mathbf{I}} + \mathbf{g} \tag{7}$$

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}\mathbf{q} \tag{8}$$

where $\mathbf{a}^{\mathbf{I}}$ is the imposed acceleration on the craft which accounts for all forces except gravity. It is expressed in the inertial frame. $\boldsymbol{\Omega}$ is given by

$$\begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix} \tag{9}$$

Here, ω_x, ω_y, and ω_z are the components of $\boldsymbol{\omega}$, the rotation rate of the vehicle with respect to the inertial frame of reference.

In terms of the measured values, we can write the dynamical system equations as

$$\dot{\mathbf{p}} = \mathbf{v} \tag{10}$$

$$\dot{\mathbf{v}} = \mathbf{L}_{Ib}(\mathbf{q})[\mathbf{a}_s - \mathbf{b}] - \mathbf{L}_{Ib}\mathbf{w}_a + \mathbf{g} \tag{11}$$

where $\mathbf{L}_{Ib}$ is the transformation matrix from the body to the inertial frame of reference in terms of the quaternions.

The dynamic system equation for the quaternions is

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega}_s - \boldsymbol{\varphi})\mathbf{q} - \frac{1}{2}\boldsymbol{\Omega}(\mathbf{w}_\omega)\mathbf{q} \tag{12}$$

For the constant accelerometer and gyro biases we have

$$\dot{\mathbf{b}} = 0 \tag{13}$$

$$\dot{\boldsymbol{\varphi}} = 0 \tag{14}$$

III. Extended Kalman Filter Implementation

For completeness, we give a brief overview of our implementation of the Extended Kalman Filter. The full derivation follows the one given in[1] closely.

Consider the equations for a nonlinear system

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t), t) + \boldsymbol{\omega}(t) && (15) \\ \mathbf{z}(t_k) &= \mathbf{h}(\mathbf{x}(t_k), t_k) + \boldsymbol{\nu}(t_k) && (16) \end{aligned}$$

where we assume that $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{h}}{\partial \mathbf{x}}$ is defined for all $\mathbf{x}$ and time t. $\boldsymbol{\omega}$ and $\boldsymbol{\nu}$ are assumed to be uncorrelated white noise processes having Gaussian distribution as before.

At the outset the reference or nominal trajectory starts from the initial condition $\hat{\mathbf{x}}(t_0) = \hat{\mathbf{x}}_0$ and satisfies the differential equation

$$\dot{\hat{\mathbf{x}}}(t) = \mathbf{f}(\hat{\mathbf{x}}(t), t) \tag{17}$$

Associated with this nominal trajactory would be the measurement equation

$$\hat{\mathbf{z}}(t_k) = \mathbf{h}(\hat{\mathbf{x}}(t_k), t_k) \tag{18}$$

Let

$$\mathbf{x} = \hat{\mathbf{x}} + \delta\mathbf{x}$$

then expanding about the nominal trajectory and dropping higher-order terms gives

$$\delta\dot{\mathbf{x}}(t) \approx \mathbf{F}[\hat{\mathbf{x}}(t), t]\delta\mathbf{x} + \boldsymbol{\omega}(t) \tag{19}$$

where

$$\mathbf{F}[\hat{\mathbf{x}}(t), t] \triangleq \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(t)} \tag{20}$$

Similarly the measurement equation can also be expanded as

$$\begin{aligned} \mathbf{z}(t_k) &= \mathbf{h}(\hat{\mathbf{x}}(t_k) + \delta\mathbf{x}, t_k) + \boldsymbol{\nu}(t_k) \\ &\approx \mathbf{h}(\hat{\mathbf{x}}(t_k), t_k) + \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}} \delta\mathbf{x} + \boldsymbol{\nu}(t_k) && (21) \end{aligned}$$

With the definition of $\hat{\mathbf{z}}(t_k)$ given in eq. (18), the linearized dynamic equation for measurement perturbations becomes

$$\mathbf{y}(t_k) = \mathbf{z}(t_k) - \hat{\mathbf{z}}(t_k) = \mathbf{H}[\hat{\mathbf{x}}(t_k), t_k]\delta\mathbf{x} + \boldsymbol{\nu}(t_k) \tag{22}$$

where

$$\mathbf{H}[\hat{\mathbf{x}}(t_k), t_k] \triangleq \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(t_k)} \tag{23}$$

The extended Kalman filter measurement update incorporates the measurement $\mathbf{z}(t_k) = \mathbf{z}_k$ by means of

$$\begin{aligned} \mathbf{K}_k &= \mathbf{P}_k^-\mathbf{H}_k^T[\mathbf{H}_k\mathbf{P}_k^-\mathbf{H}_k^T + \mathbf{R}_k]^{-1} && (24) \\ \hat{\mathbf{x}}_k^+ &= \hat{\mathbf{x}}_k{}^- + \mathbf{K}_k[\mathbf{z}_k - \mathbf{H}_k(\hat{\mathbf{x}}_k{}^-)] && (25) \\ \mathbf{P}_k^+ &= \mathbf{P}_k^- - \mathbf{K}_k\mathbf{H}_k\mathbf{P}_k^- && (26) \end{aligned}$$

where $\mathbf{H}$ is given by eq. (23). Here, $\mathbf{R}$ is the covariance of the sensor readings. The estimate is propagated forward to the next sample time t_{k+1} using the equations

$$\begin{aligned} \hat{\mathbf{x}}_{k+1}^- &= \hat{\mathbf{x}}_k^+ + \int_{t_k}^{t_{k+1}} \mathbf{f}(\hat{\mathbf{x}}_k^+, t)\ dt && (27) \\ \mathbf{P}_{k+1}^- &= \phi_k\mathbf{P}_k^+\phi_k^T + \mathbf{Q}_k && (28) \end{aligned}$$

where ϕ is the state transition matrix defined by

$$\dot{\phi} = \mathbf{F}\phi; \quad \text{where} \quad \phi(t_k) = \mathbf{I} \tag{29}$$

Note that we have, as in[1,2] and others, chosen to use the states themselves as our filtered variables. A second approach, often used in extremely high-accuracy implementations, is to write the filter in terms of the deviations from the nominal trajectory,[3] and add these corrections to the propagated nominal.

III.A. The Vision Based Update

We use the standard pin-hole camera model, as in.[4] While it is possible in theory to derive full state information from vision alone,[4–6] some of the states are much harder to estimate well. More success is likely when at least some additional information is available.[6]

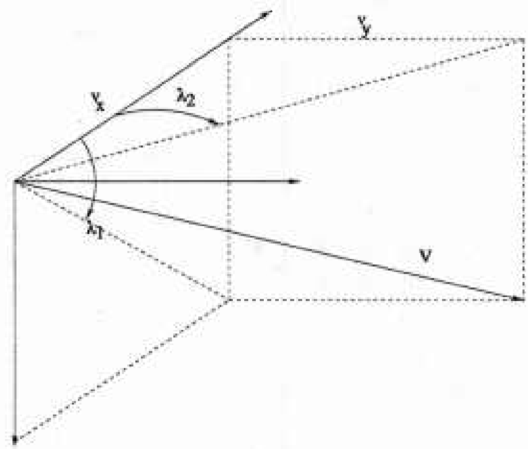


Figure 1. Pinhole camera model

In this work, we have chosen instead to assume that only one measurement is asked of the vision sensor. It is easily shown[5] that direction of flight of the vehicle is separable from other apparent motion of objects in the camera field of view. We assume that this measurement is delivered to the navigation filter.

The velocity direction in the body frame is described by two angles as shown in figure 1. We can see that

$$\lambda_1 = \tan^{-1}\left(\frac{v_z^b}{v_x^b}\right) \tag{30}$$

$$\lambda_2 = \tan^{-1}\left(\frac{v_y^b}{v_x^b}\right) \tag{31}$$

Now, we can convert the inertial velocity into the body as

$$\begin{bmatrix} v_x^b \\ v_y^b \\ v_z^b \end{bmatrix} = \mathbf{L}_{bI}(\mathbf{q})(\mathbf{v})^I \tag{32}$$

(the superscripts I and b denote the respective frames of reference). We thus have the two measurements λ_1 and λ_2 in terms of the states $\mathbf{v}^I$ and $\mathbf{q}$. We use this in equations (30) and (31), and taking the appropriate derivatives provides the measurement coefficient matrix associated with the vision update, as in eqn. (23).

III.B. Propagation between GPS Measurements

When there is no GPS measurement, but there is an inertial measurement, we propagate the transition matrix ϕ and the *a priori* estimate $\bar{\mathbf{x}}$. Let the measurement be taken at time t_k and let $t_{k+1} = t_k + \Delta t$ where Δt is a known increment.We approximate the solution to the dynamical system equations as

$$\begin{aligned}
\bar{\mathbf{p}}_{k+1} &= \bar{\mathbf{p}}_k + \bar{\mathbf{v}}_k \Delta t && (33)\\
\bar{\mathbf{v}}_{k+1} &= \bar{\mathbf{v}}_k + [(\mathbf{L}_{Ib}(\bar{\mathbf{q}}_k)[\mathbf{a}_{s,k} - \bar{\mathbf{b}}_{\mathbf{k}}]) + \mathbf{g}]\Delta t && (34)\\
\bar{\mathbf{q}}_{k+1} &= \bar{\mathbf{q}}_k + \frac{1}{2}\Omega(\boldsymbol{\omega}_{s,k} - \bar{\varphi}_k)\bar{\mathbf{q}}\Delta t && (35)\\
\bar{\mathbf{b}}_{k+1} &= \bar{\mathbf{b}}_k && (36)\\
\bar{\varphi}_{k+1} &= \bar{\varphi}_k && (37)
\end{aligned}$$

Here $\mathbf{a}_s$ and $\boldsymbol{\omega}_s$ are the measured accelerations and rotations respectively. Now the *process noise covariance* matrix is propagated as shown below. The value of ϕ is approximated as

$$\phi(t_{k+1}, t_k) \approx \mathbf{I} + \mathbf{F}_k \Delta t + \frac{1}{2}\mathbf{F}_k^2 \Delta t^2 \tag{38}$$

The matrix $\mathbf{F}$ is computed each time an inertial measurement is available.This matrix can be written in block form as

$$\begin{bmatrix} 0 & \mathbf{I}_3 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{F}_{23} & \mathbf{F}_{24} & 0 \\ 0 & 0 & \mathbf{F}_{33} & 0 & \mathbf{F}_{35} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{39}$$

where using eq. (10) - eq. (14) we have

$$\begin{aligned}
\mathbf{F}_{23} &= \frac{\partial}{\partial \mathbf{q}}\left[\mathbf{L}_{Ib}(\mathbf{q})(\mathbf{a}_s - \mathbf{b})\right]\\
\mathbf{F}_{24} &= -\mathbf{L}_{Ib}(\mathbf{q})\\
\mathbf{F}_{33} &= \frac{1}{2}\Omega(\boldsymbol{\omega}_s - \varphi)\\
\mathbf{F}_{35} &= \frac{1}{2}\frac{\partial}{\partial \varphi}(\Omega(\boldsymbol{\omega}_{\mathbf{s}} - \varphi)\mathbf{q})
\end{aligned}$$

III.C. Pseudorange Rate Update

The preceding development assumes a full solution from the GPS receiver. He requires that a minimum of four satellites be in view however, which may not always be the case. Also, there are likely to be other reason why we would wish to use the pseudoranges and rates directly, rather than the receiver solution.

Let the unit vector, $\mathbf{u}$ in the direction of a satellite in inertial space (NED frame) be

$$\mathbf{u} = u_N \hat{\mathbf{i}} + u_E \hat{\mathbf{j}} + u_D \hat{\mathbf{k}}$$

where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are the respective unit vectors in the NED frame. Since we can express u_N, u_E, u_D in terms of the azimuth and elevation angles we have the unit vector in the direction of that satellite in terms of known quantities. We then take the dot product of this unit vector with the velocity estimate

$$\mathbf{ps}_{rate} = \mathbf{v}^I \cdot \mathbf{u}$$

Hence $\mathbf{ps}_{rate}$ gives us an estimate of the pseudorange rate in the direction of that particular satellite in inertial space. This when compared to the measured value of the pseudorange rate from the GPS receiver, gives us the error which drives the filter. Now taking the appropriate derivatives provides the measurement coefficient matrix associated with the pseudorange rate update, as in eqn. (23).

IV. Simulation Runs:

Data used to verify the traditional GPS/INS fusion filter with the vision update was simulated using a dynamic model of an aircraft and sensor model. The IMU data comes in at 100 Hz, the vision data comes in at 20 Hz and the GPS data comes in at 1Hz. We have a state estimate and its associated error variance coming out of the filter at 100 Hz.

Similarly while verifying the filter based on the pseudorange rate update we bring in the pseudorange rate information at 1Hz. The IMU and the vision information comes in at the same rate as mentioned in the paragraph above.

IV.A. Choice of the Sensor Specifications:

The covariances for the GPS have been chosen from a variety of sources to represent the typical values for such instruments. The covariances used in the INS were based on the specifications provided in the reference.[7]

Process Noise Covariance Q: Q is taken to be a diagonal matrix with all entries zero other than those associated with the accelerometers and gyros. The accelerometer covariance is taken to be 0.0144 m/s^2 in all channels. The gyro noise covariance is assumed to be 0.000144 1/s.

GPS Noise Covariance: The diagonal elements (variances) of the sensor noise covariance matrix used here are

$$[4.0, 4.0, 49.0, \quad 0.01, 0.01, 0.01]$$

with units of meters and meters per second, as appropriate.

Vision Sensor Noise Covariance: The diagonal elements of the vision noise covariance matrix used are

$$[0.0049,\ 0.0049] \text{ radians.}$$

Pseudorange Rate Noise Covariance: While verifying the filter based on the pseudorange rate update the diagonal elements (variances) of the sensor noise covariance matrix used are

$$[0.1,\ 0.1,\ 0.1] \text{ m/s.}$$

IV.B. Results:

IV.B.1. Filter with GPS and Vision Update.

The plot in figure 2 represents the three dimensional position history of the truth that we are tracking. The aircraft banks to about ninety degrees when taking the two sharp left turns, one when the y coordinate is about -500 m and the next when the y coordinate is about -4000 m. (A third turn appears near the end of the data set used.) Since the data to be processed is simulated inflight data, the first data set is taken to be the initial conditions for the sensors.

Shown below are the time histories of a couple of the filter states along with their associated error variance. These plots are based on the data processed by the GPS/INS fusion filter with vision update. In these plots, the dotted lines show the history without the vision update whereas the solid line shows the history with the vision update.

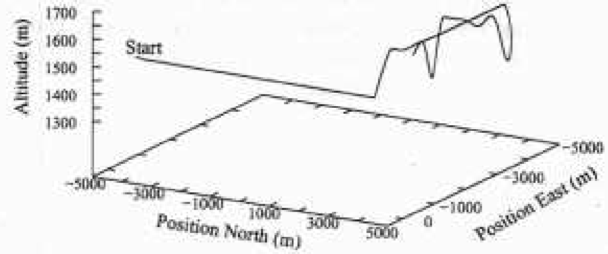
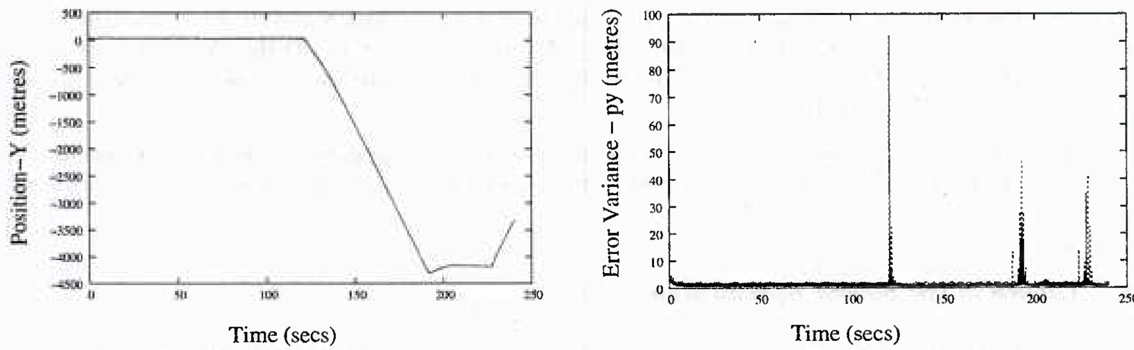


Figure 2. Three dimensional motion

IV.C. Lateral Position



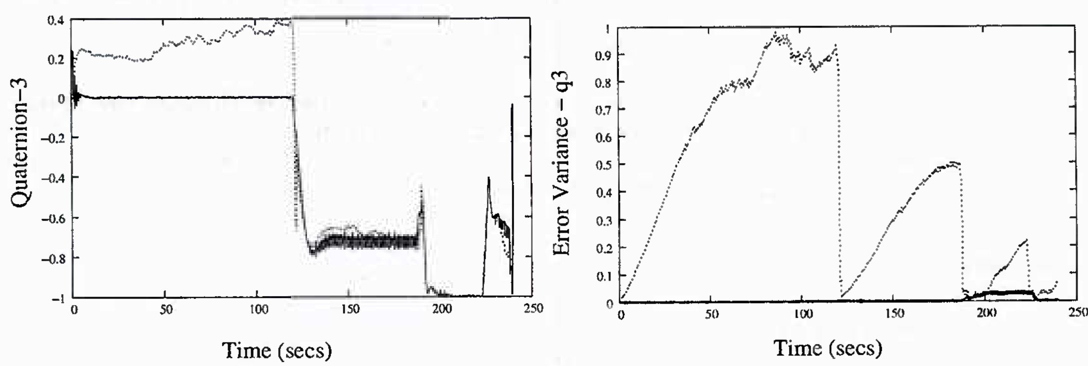
(a) Lateral position

(b) Variance estimate

Figure 3. Lateral position estimate and estimated variance.

Figure 3(a) and shows the estimated lateral position. The estimates with the vision update and the ones without the update virtually superpose. The left turns taken by the craft occur at around $t = 125$ *sec* and around $t = 190$ *sec*. In figure 3(b) notice the fact that the excursion in the error variance history without the vision update of about $90m$ at $t = 125$ *sec* and $50m$ at $t = 190$ *sec* are not seen in the error variance with the vision update. Similar behavior was seen in the other position variables.

IV.D. Quaternion-3



(a) Quaternion-3

(b) Error-q3

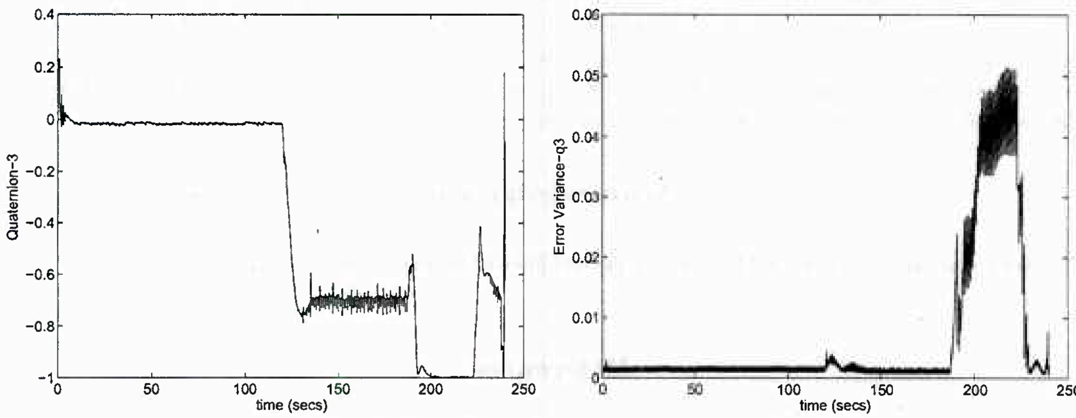
Figure 4. Estimate and variance of quaternion q_3

The quaternions describe the orientation of the aircraft in the local frame. Here we see distinct differences between the plots for the estimates with the vision update and the one without it. The estimate is nearly exact during the steady portion of the flight, reflecting the additional information from the vision sensor.

In figure 4(b) we have the estimated error variance for the third quaternion. Unsurprisingly, the variance grows during the steady phases of the flight when there is no vision update. The additional information from the vision sensor makes the angular states fully observable, however, and the variance estimate remains nearly zero when it is included.

IV.E. Filter with Pseudorange Rate and Vision Update.

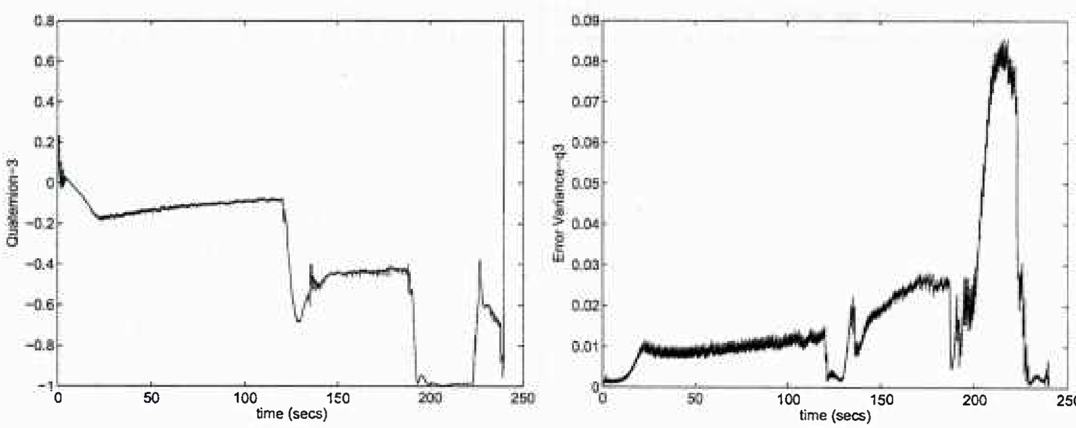
In the situation when we don't have a GPS solution and are unable to provide the GPS update to the filter, we can use pseudorange rate information from the GPS receiver to update the filter until we have full GPS solution again. Investigation in that direction yields (as expected) that we need pseudorange rate information from a minimum of three satellites to arrive at a navigation solution of useful accuracy.



(a) Quaternion-3 (3 satellites)　　(b) Error-q_3 (3 satellites)

Figure 5. Estimate of q_3 using pseudorange from 3 satellites.

As an illustration, we give here the results for the third quaternion. In this case, we have used only pseudorange rate, without including pseudorange, as the immediate goal of our work is primarily concerned with orientation angles. Figure 5(a) shows the estimate with 3 satellites in view. Comparison with Figure 4(a) shows excellent agreement with earlier results, and in fact the estimate is close to truth. The estimated variance is however much smaller.



(a) Quaternion-3 (2 satellites)　　(b) Error-q_3 (2 satellites)

Figure 6. Estimate of q_3 using pseudorange from 2 satellites.

Just to illustrate what happens when we have only two satellites in view, figure 6(a) illustrates the same estimate in such a situation and figure 6(b) illustrates the associated error variance estimate. The estimate is not as good, especially during the steady cruise phase at the beginning of the trajectory. It dials in quite well during the later maneuvering stages, however. The estimated variance indicates less confidence in the estimate. Formal observability analysis has not yet been done in these cases.

V. Conclusion

In this paper the traditional GPS/INS fusion filter was successfully extended to include a velocity direction update from a vision sensor, and the performance of the augmented filter was investigated. Results show that the vision information successfully improves the estimates of the state variables and lowers the associated error covariances. Further work in this direction using pseudorange rate data update instead of the traditional GPS update shows us that the filter performs satisfactorily with pseudorange rate data from a minimum of three satellites, and surprisingly well with data from only two. This could potentially lead to the filter being robust to the loss of GPS, at least for a short period of time. The results also indicate that the direct use of pseudorange rate information (as opposed to a velocity solution from the GPS receiver) might be advisable for the estimation of orientation angles.

Acknowledgment

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